

Handout 2: Sets, summations, and for statements

1 Examples

1. Note that

$$3 + 3 + 3 + 3 = \sum_{i=1}^4 3 = 3 \sum_{i=1}^4 1.$$

2. Also, note that

$$1 + 2 + 3 + 4 + 5 + 6 = \sum_{i=1}^6 i \quad \text{and} \quad 2 + 4 + 6 + 8 + 10 = \sum_{i=2}^5 2i = 2 \sum_{i=1}^5 i = 10 + 2 \sum_{i=1}^4 i.$$

3. Let $T = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. Then

$$\sum_{i \in T} i = 0 \quad \text{and} \quad \sum_{i \in T} |i| = 20.$$

4. Let $T = \{3, 6, 8, 14\}$. Then

$$\sum_{i \in T: i \geq 7} i = 8 + 14 = 22.$$

5. Let $a_1 = 2$, $a_2 = 5$, and $a_3 = 7$. Let x_1 , x_2 , and x_3 denote decision variables. Then

$$\sum_{i=1}^3 a_i x_i = 2x_1 + 5x_2 + 7x_3.$$

6. Consider the following inequalities.

$$\begin{aligned} x_1 + 2x_2 - 4x_3 + 4x_4 &\leq 5 \\ 2x_1 + 4x_2 + x_4 &\leq 7 \\ 5x_1 - x_2 + 5x_3 + 2x_4 &\leq -2 \end{aligned}$$

Define $R = \{1, 2, 3\}$ and $C = \{1, 2, 3, 4\}$. For $i \in R$ and $j \in C$, let a_{ij} denote the coefficients of the variables x_j in the inequalities and b_i denote the right-hand sides. This means that

$$\begin{aligned} a_{11} &= 1, a_{12} = 2, a_{13} = -4, a_{14} = 4, \\ a_{21} &= 2, a_{22} = 4, a_{23} = 0, a_{24} = 1, \\ a_{31} &= 5, a_{32} = -1, a_{33} = 5, a_{34} = 2, \end{aligned}$$

and

$$b_1 = 5, b_2 = 7, b_3 = -2.$$

Then, the inequalities can be succinctly expressed as

$$\sum_{j \in C} a_{ij} x_j \leq b_i \quad \text{for } i \in R.$$

2 Homework problems

- Using a summation, write an expression summing the first 20 positive integers. Then use a summation to write

$$3 + 6 + 9 + 12 + 15.$$

- Define indexed parameters and use a summation to express the following sum over the decision variables y_1, y_2, y_3, y_4 .

$$9y_1 - y_2 + 3y_3 + 11y_4.$$

- Define sets and indexed parameters to express the following inequalities using a single for statement and a summation over the decision variables z_1 and z_2 .

$$z_1 + z_2 \geq 1$$

$$z_1 \geq 3$$

$$z_2 \geq 4$$

$$2z_1 + 4z_2 \geq 7$$

$$3z_1 + 7z_2 \geq 9$$

- Assume the index sets R and S have been defined. Also, for $i \in R$ and $j \in S$, assume that the parameters a_{ij} and b_i have been defined as well as the decision variables x_j . Some of the following expressions have indexing errors. Identify what they are or indicate the expression is correct.

(a) $\sum_{j \in S} a_{ij} x_j \leq b_i$ for $i \in R$ and $j \in S$.

(b) $\sum_{i \in S} a_{ij} x_j$ for $j \in R$.

(c) $\sum_{j \in S, i \in R} a_{ij} x_j \leq b_i$.

(d) $\sum_{i \in S} a_{ji} x_i \leq b_j$ for $j \in R$.